12. N. V. Berdinskaya and M. M. Kusakov, "Experimental study of the mechanism of liquid transfer from the film to capillary-contained state in the capillary pore model," Kolloidn. Zh., 34, No. 5, 755-760 (1972).
13. B. V. Deryagin and M. M. Kusakov, "Experimental study of solvate surfaces," Izv. Akad. Nauk SSSR, Ser. Khim., No. 5, 1119-1122 (1937).
14. B. V. Deryagin and S. V. Nerpin, "Equilibrium, stability, and kinetics of free liquid films," Dokl. Akad. Nauk SSSR, 99, No. 6, 1029-1032 (1954).

COLLECTIVE EFFECTS IN A DENSE SYSTEM OF LARGE BUBBLES

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UDC 532.529

The rising velocity and interphase transport of a dense bubble swarm in a homogeneous liquid are investigated, along with the motion and external mass transfer in an inhomogeneous porous medium.

Under confined-flow conditions the interaction of bubbles forming a dense swarm significantly alters their hydrodynamic and mass-transfer characteristics in comparison with solitary bubbles. The values of the Reynolds number, which characterizes the flow around solitary bubbles, are usually large in the majority of real situations, so that the liquid flow in the space between bubbles can be regarded as inviscid and potential everywhere except in thin boundary layers on the bubble surfaces and in their hydrodynamic wake regions. In this article we investigate the two extreme cases in which the influence of surface tension is very strong and very weak and, accordingly, the bubbles approach the configurations of a sphere and a spherical cap.

The motion of solitary bubbles of the first type was first studied theoretically in [1, 2], and their mass transfer with the surrounding medium in [3]. A hydrodynamic model of bubbles of the second type was first proposed by Davies and Taylor [4] and was later refined by Parlange [5]; their mass transfer with the surrounding medium has been investigated in [6, 7], and bubbles of this type have been investigated more in detail in [8, 9]. Collective effects have apparently been studied only for spherical bubbles on the basis of the well-known cell model [10, 11]. Below, we investigate these effects by means of the powerful machinery of ensemble averaging and the methods of self-consistent field theory (see [12] and the survey [13]).

Filtration and External Mass Transfer in an Inhomogeneous Porous Solid

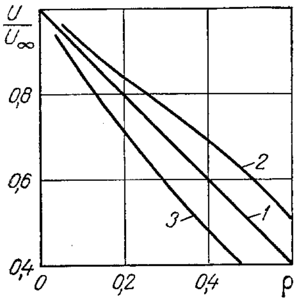
We first consider the auxiliary problem of determining the effective permeability of an inhomogeneous porous solid comprising a macroscopically homogeneous porous medium ("matrix") and discrete porous inclusions, which are also macroscopically homogeneous and are distributed in the matrix. The Darcy equations

$$\mathbf{Q}_i = -(k_i/\mu)\nabla P_i, \quad \operatorname{div}\mathbf{Q}_i = 0 \tag{1}$$

for the local values of the filtration rate **Q** and the pressure P are valid in the matrix and in the inclusions. At the boundaries of the inclusions the pressure and the normal component of the flow velocity are continuous. It is required to determine the "effective" permeability k, i.e., the coefficient in the equation $\mathbf{q} = -(k/\mu)\nabla p$, which relates the mean filtration rate to the pressure in the inhomogeneous solid (in the simplest case **q** and p can be interpreted as the results of averaging **Q** and P over a "small" physical volume containing a sufficiently large number of inclusions).

The statement of this problem is identical to the statement of the problem of determining the effective thermal conductivity of a composite material if the pressure is identified

Ural State University, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 6, pp. 1057-1066, December, 1981. Original article submitted November 30, 1980.



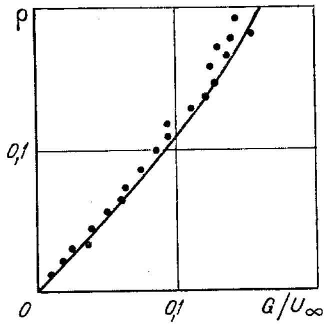


Fig. 1 Fig. 2

Fig. 1. Relative rising velocity versus ρ for spherical and cap ($\rho' \ll \rho$) bubbles. 1) Model of a moderately dense system; 2) model with a concentric layer of pure liquid; 3) Marrucci equation [10].

Fig. 2. Comparison of the theoretical equation (14) for a moderately dense system with the experimental results of [23] on the rising of air bubbles in a vertical column filled with water.

with the temperature, the filtration rate with the heat flux, and the ratios k_i/μ and k/μ with the thermal conductivities of the matrix, inclusions, and the material itself as a unit whole. We can therefore write at once

$$k = \beta(\varkappa, \rho) k_0, \quad \varkappa = k_1/k_0, \tag{2}$$

where the function β has the same form as in the heat-conduction problem. Within the framework of the notions and method of [12] this quantity is determined by analyzing the special problem of an isolated ("test") inclusion immersed in a fictitious medium, whose thermal conductivity (or permeability) at a particular point depends on the position of the point relative to the inclusion, the nature of this dependence being determined by the characteristics of the spatial distribution of all the inclusions. For spherical particles β has been calculated without regard for the mutual impenetrability of the spheres (i.e., for a homogeneous fictitious continuum), with approximation of the true dependence of the properties of the fictitious medium on the distance to the surface of the test sphere by elementary power functions [i.e., for a homogeneous fictitious medium separated from the test sphere by a concentric (with the latter) layer having the same properties as the original matrix], and for various forms of the binary distribution function of the spheres. In particular, the functions obtained by Kirkwood, Percus, and Yevick have been investigated, along with the function corresponding to the hypothesis of a uniform distribution of centers of the spheres in the region exterior to a sphere centered at the same point as the test sphere, but with twice the radius [14, 15]. The analogous problem for systems containing cylindrical inclusions has been solved in [16]. It is clear that the results of [14-16] are directly applicable to the immediate problem of the effective permeability.

The dependence of β on its arguments and certain analytical expressions for the average fields in the vicinity of the test inclusion are given in [14-16]. Here we give expressions for β and the tangential component of the filtration rate in the matrix at the surface of a test inclusion in the case where all the inclusions are impenetrable spheres. These expressions are needed below in order to study a system of bubbles. The model of an inhomogeneous medium with a moderate density of inclusions (disregarding the mutual impenetrability of the spheres) is characterized by the relations

$$\beta = \beta(0, \rho) = 1 - \frac{3}{2}\rho, \quad w_s = \frac{q_s^-}{1-\rho} = \frac{3}{2} SU \sin\theta, \quad S = (1-\rho)^{-1}, \tag{3}$$

and the approximate model of a very dense medium, in which a homogeneous layer with the properties of the matrix is introduced at the surface of the test sphere, corresponds to

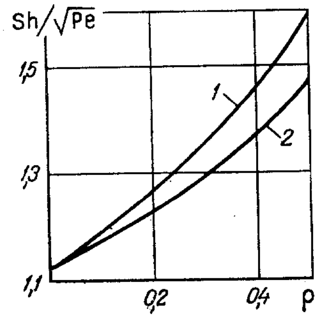


Fig. 3. Relations (21) and (22) for $\rho'/\rho \ll 1$. 1) Model of a moderately dense bubble system; 2) model with a concentric layer of pure liquid.

$$\beta=\beta(0,\ \rho)=\frac{1}{17+7\rho}\{5-11\rho+[(5-11\rho)^2+7(1-\rho)(17+7\rho)]^{1/2}\},$$
$$w_s=\frac{q_s}{1-\rho}=\frac{3}{2}SU\sin\theta,\quad S=\frac{2}{3}\left(\frac{1}{1-\rho}+\frac{7\beta+5}{17\beta+7}\right), \tag{4}$$

where $U=\beta(k_0/\mu)E$ is the average uniform volume flow rate in the inhomogeneous porous solid, corresponding to a uniform pressure gradient $\nabla p=-E$, and θ is the polar angle in a coordinate system attached to the test sphere (the polar axis is directed upstream).

The function S is introduced in (3) and (4) so that the dependence of the surface velocity on SU under confined-flow conditions will coincide with its dependence on U_∞ in flow around a solitary sphere. On the basis of the results of [14-16] this function is readily determined for other models of a system of spheres as well as for media containing cylindrical inclusions.

The problems of transport toward a solitary impenetrable sphere and cylinder immersed in an infiltrable porous medium have been solved for small and large Péclet numbers [17]. The various results in [17] correspond to final relations of the form

$$\mathrm{Sh}_\infty=f(\mathrm{Pe}_\infty,\ \gamma_\infty),\quad \mathrm{Pe}_\infty=\frac{2RU_\infty}{D},\quad \gamma_\infty=\frac{lU_\infty}{D}, \tag{5}$$

where the parameter γ describes the relative contribution of convective dispersion associated with mixing of elementary streams in the intercepted pore space, and the effective molecular diffusivity in the porous medium D is determined in this case with regard for the sinuosity factor.

For small Péclet numbers the mass-transfer process between any inclusion and the flow is determined by the state of the entire porous medium surrounding the inclusion. Under confined-flow conditions the effective convective transfer coefficient and diffusion in this medium are determined by the quantities U and βD in exactly the same way as they are determined by U_∞ and D in flow around a solitary inclusion. Therefore, the parameter Sh for confined flow around many spheres or cylinders is described by the same functions as those involved in (5) if $2RU/\beta D$ and $lU/\beta D$ are taken as their arguments (since $l \ll R$, the second quantity can be set equal to zero in this case).

For large Péclet numbers the main resistance to mass transfer is concentrated in a thin diffusion boundary layer at the surface of the inclusion. The diffusivity in this case is clearly independent of the presence of other inclusions, but the velocity in the layer is determined, according to (3), (4), or other analogous relations, by the quantity SU, rather than by U_∞. Consequently, Sh is once again expressed in the form (5), but now the arguments of the functions f are $2RSU/D$ and lSU/D.

The foregoing analysis can therefore be used to extend the results of [17] for the mass transfer of solitary spheres and cylinders to large systems of such bodies. If the conductance through the skeleton of the porous matrix is small (as is customarily observed for granular layers), then all of the preceding conclusions are equally valid for external heat transfer in an infiltrable porous medium. We note that the problems discussed here are also of considerable independent significance in relation to the requirements of filtration theory, the mechanics of saturable soils, petroleum-refining practice, chemical technology, etc.

Bubble Rising Velocity

We now consider confined liquid flow in a macroscopically homogeneous swarm of bubbles of equal size, using a coordinate system in which the bubbles are at rest on the average. When the shape of the bubbles is close to spherical and the flow in the spaces between them is close to potential, the statement of the problem of determining this flow coincides formally with the statement of the problem discussed above if we now interpret Q_0 and $-P_0$ as the local values of the velocity and potential of the liquid and assume that $k_0/\mu = 1$ and $k_1/\mu = 0$ [the quantities Q_1 and P_1 in this case do not have direct physical significance and are regarded simply as the solutions of Eqs. (1); of course, they do not describe the actual gas flow in the bubble interiors]. The quantity U represents the uniform average rate of liquid filtration in a swarm of impenetrable bubbles. If the bubble rising velocity in a coordinate system in which the liquid is at rest on the average is equal to U_b, then clearly

$$U_b = U/(1-\rho). \tag{6}$$

The force of hydrodynamic interaction of any bubble of the swarm with the liquid flow under the conditions of practically nonseparating flow around the bubble is expressed in the usual way as the integral, over the spherical surface of the bubble, of the density of forces exerted on it by the stresses in the liquid [18]. This force can be written as the sum of the effective buoyancy and the force $\mathbf{F}^*$ produced only by viscous stresses, i.e.,

$$\mathbf{F} = \mathbf{F}^* - V d\mathbf{g}. \tag{7}$$

If the shape of the bubble does not depend on the presence of other bubbles in the system, then the variation induced by the latter in the hydrodynamic environment of the given bubbles does not affect the shape and structure of the thin boundary layer at its surface, but is felt only in the characteristic value of the velocity at the outer boundary of the layer; this characteristic velocity is proportional to w_s and determines the strain rates created in the layer. Inasmuch as the viscosity in the indicated layer does not depend on the existence of the other bubbles, the viscous stresses in it are governed by the quantity $w_s = SU$, to which they are proportional. If the flow around the bubbles is almost nonseparating, the viscous drag experienced by them will be determined entirely by the indicated stresses, i.e., we can put $F^* = KSU$, where the coefficient K in this case, based on the foregoing discussion, can be considered to be independent of the presence of the other bubbles (i.e., of the quantity ρ) and is determined from the relation $F^*_\infty = KU_\infty$ for a solitary bubble. On the basis of (6) we have

$$U/U_\infty = F^*/SF^*_\infty, \quad U_b/U_\infty = F^*/(1-\rho)\, SU_\infty, \tag{8}$$

so that to determine the dependence of U_b on ρ it is necessary to find the analogous dependence for F^*.

The customary system of phenomenological equations of motion of the phases of a disperse system [19] in the case of steady flow along the vertical coordinate z yields the equations (the z axis is directed opposite to g)

$$-(1-\rho)\frac{\partial p}{\partial z} - (1-\rho)\, d_0 g + nF^* = 0, \quad -\rho\frac{\partial p}{\partial z} - \rho d_1 g - nF^* = 0. \tag{9}$$

Hence we obtain

$$\partial p/\partial z = -[(1-\rho)\, d_0 + \rho d_1]\, g, \quad nF^* = \rho(1-\rho)(d_0 - d_1)\, g \tag{10}$$

and then

$$U_b/U_\infty = 1/S. \tag{11}$$

It has been shown in [12] for suspensions and discussed, for example, in [20] that Eqs. (9) can also be written in the form

$$-\frac{\partial p}{\partial z} - (1-\rho)\, d_0 g + nF = 0, \quad -\rho d_1 g - nF = 0, \tag{12}$$

from which, on the basis of (8), we obtain the previous expressions (10) and (11).

For small values of ρ we can take $U_b/U_\infty = 1 - \alpha\rho$, where the models underlying (3) and (4) correspond to values of α equal to unity and $35/48 \approx 0.73$, respectively.

We now mention a frequently perpetrated error. According to Nicklin [21], under the conditions of continuous bubble generation the rising velocity in a vertical column in which the liquid is at rest on the average must exceed the above-calculated velocity U_b by the amount of the volume flow rate of the gas G, whence it follows that $G = \rho(U_b + G)$, i.e., $G = \rho U/(1 - \rho)$, so that the observed rising velocity should satisfy, rather than (11), the relation

$$\frac{U_b}{U_\infty} = \frac{1}{(1-\rho)S}. \qquad (13)$$

The arguments of [21] are valid in the case of the ascent in a column of a string of large "pistons" filling up the entire cross section of the column, in which case the average velocity of the liquid in cross sections situated above and below each piston wholly within the liquid is in fact nonzero and equal to G. But these arguments break down in bubble flow, where the average liquid flow across any cross section of the column continuously intercepted by the bubbles is equal to zero. The fallacy of these arguments in application to the case under investigation and of the expression (13) based on them has been noted previously in [22] (see the footnote on p. 418).

The dependences of U/U_∞ on ρ for the models underlying (3) and (4) are shown in Fig. 1. Also shown is the dependence obtained by Marrucci [10] on the basis of a calculation of the dissipation in the unit cell around a spherical bubble. This dependence describes the asymptotic behavior as the Reynolds number tends to infinity, subject to the condition that the bubble preserves sphericity. The analogous curves for large but finite values of the Reynolds number lie somewhat below the one shown here [11].

For the volume flow rate of the disperse phase as a function of its volume concentration in a coordinate system in which the dispersion is at rest on the average, we obtain from (11)

$$\frac{G}{U_\infty} = \frac{\rho}{S}. \qquad (14)$$

This equation describes, for example, the ascent of a system of continuously generated bubbles in a column or the situation in a percolation layer. For the model of a moderately dense system $[S = (1 - \rho)^{-1}]$ expression (14) coincides with the empirical formula verified experimentally in [19], in which it has been shown to be in good agreement with the experimental data of S. S. Kutateladze and V. N. Moskvicheva on the percolation of water drops in a mercury bath. Figure 2 shows the results of a comparison between the theory and the experiment in [23] on the rising of air bubbles in a column filled with water or a water–glycerin mixture.

We can readily obtain expressions corresponding to (11) and (14) in the case where the average velocity of the liquid has a nonzero value. For example, in the rising of a bubble swarm that is bounded above and below and fills up the entire cross section of the column, the fluid, which is at rest on the average outside the swarm, moves downward through it with a velocity $U^* = G/(1 - \rho)$, referred to the total cross section of the column. In this case $G = \rho(U - U^*)$, i.e., in place of (14) we now have

$$\frac{G}{U_\infty} = \frac{\rho(1-\rho)}{S}. \qquad (15)$$

The analogous formula with $S = (1 - \rho)^{-1}$ has also actually been verified in [19] by comparison with the experimental data of Shulman and Molstedt.

We emphasize the fact that these results apply only to systems of spherical bubbles distributed at random in space, in which case it is permissible to neglect, first, the variations of the bubble shape with increasing confinement of the flow (it is expected on the basis of general considerations that the bubbles in this case will be better "streamlined," corresponding to a certain reduction of the coefficient K with increasing ρ) and, second, any possible structuring of the system, for example by the entrainment of some bubbles in the wake zones of others. Both of these neglected effects will necessarily promote a certain increase in the velocity of confined bubbles rising in comparison with that calculated above. They are conceivably the factors responsible for such an increase observed in certain experiments [19].

We now discuss bubbles having the shape of a spherical cap, restricting the problem to the case of laminar flow around the bubbles, which are followed by domains with closed rotational motion of the liquid filling them. The shape of each domain together with the bubble itself approaches sphericity [4, 5, 18], while the flow in the spaces between these spheres is potential and is identical with the flow between spherical bubbles discussed above.

To determine the rising velocity under confined-flow conditions we use the classical method of Davies and Taylor [4]. Along a streamline in a thin layer next to the surface of a sphere

$$d_0 w_s \frac{\partial w_s}{\partial s} = -\frac{\partial p_s}{\partial s} + d_0 g_s, \tag{16}$$

where s is the coordinate measured along the streamline and g_s is the projection of **g** onto its direction. Making use of the fact that the pressure in the bubble interior is constant in the first approximation, we arrive at the Bernoulli equation in the form

$$w_s^2/2 = gR(1-\cos\theta). \tag{17}$$

Using (3) and expanding the right-hand side of (17) into a power series in the small quantity $\sin^2\theta$, we see that for (17) to be satisfied in the vicinity of the point of flow impingence the following equality must hold:

$$\frac{U}{U_\infty} = \frac{1}{S}, \quad U_\infty = \frac{2}{3}(gR)^{1/2}. \tag{18}$$

However, in a system of cap bubbles the quantity ρ has the significance of the volume concentration of spheres comprising both the bubbles themselves and the corresponding rotational domains, and they greatly exceed (by roughly an order of magnitude) the volume concentration ρ' of the gaseous phase. Accordingly, the quantity U represents the filtration rate of liquid through a swarm of such spheres and is not related to the rising velocity U_b of the bubbles in a coordinate system in which the liquid is at rest on the average by Eq. (6). In a coordinate system in which only the liquid outside the sphere is at rest on the average and the bubbles rise with the velocity $U' = U/(1-\rho)$, the average velocities of the gas and the liquid over the cross section are equal to $\rho' U'$ and $(\rho-\rho')U'$. Consequently, $U' = U_b + (\rho-\rho')U'$, and

$$\frac{U_b}{U_\infty} = [1-(\rho-\rho')]\frac{U'}{U_\infty} = \frac{1-(\rho-\rho')}{1-\rho}\frac{U}{U_\infty} = \frac{1-(\rho-\rho')}{(1-\rho)S}. \tag{19}$$

For $\rho' \ll \rho$ this relation practically coincides with (11).

Mass Transfer of Bubbles

The parameters characterizing the mass transfer of bubbles in a dense swarm with liquid flowing around them are readily determined by arguments of the same type as were used above in analyzing the external mass transfer in a infiltrable medium. The large bubbles investigated here are typified by large Péclet numbers, and so it suffices to consider only the asymptotic relations for $\mathrm{Pe} \gg 1$.

For a solitary spherical bubble we have [3]

$$\frac{\mathrm{Sh}_\infty}{\sqrt{\mathrm{Pe}_\infty}} = \frac{2}{\sqrt{\pi}}\left(1-\frac{2{,}89}{\sqrt{\mathrm{Re}_\infty}}\right) \approx 1.13, \quad \mathrm{Re}_\infty \gg 1. \tag{20}$$

Under confined-flow conditions the diffusivity in a thin diffusion boundary layer at the surface of the bubble remains the same as in flow around a solitary bubble; it is equal to the molecular diffusivity. However, the velocity of the liquid in this layer varies in accordance with expressions (3) and (4). Therefore,

$$\mathrm{Sh}/\sqrt{\mathrm{Pe}_b} \approx 1.13\sqrt{S}, \tag{21}$$

where Pe_b is determined relative to the velocity U_b. Relations (21), which correspond to the models (3) and (4), are shown in Fig. 3. The first coincides with the relation proposed earlier in [11]. It is instructive to compare Sh with the number Sh_∞ for natural bubble ascent in the field of gravity. Using (11), from (20) and (21) we obtain $\mathrm{Sh} = \mathrm{Sh}_\infty$.

For a solitary bubble in the shape of a spherical cap, assuming that mass transfer takes place only through its curved frontal surface, we again have the expression $Sh_\infty/\sqrt{Pe_\infty} = C$, where the constant C depends on specifically what area (the total bubble surface or only its frontal part) and what dimension (R or the equivalent radius) are involved in the determination of Sh and Pe, as well as on the maximum value of θ, which governs the position of the sharp edge of the cap on the total sphere [6, 7]. The latter quantity uniquely determines the relationship between ρ and ρ', and data pertaining to its dependence on R may be found in [5]. Under confined-flow conditions, together with (21), we obtain the following by the same arguments as before:

$$\frac{Sh}{\sqrt{Pe_b}} = C\left[S\,\frac{1-\rho}{1-(\rho-\rho')}\right]^{1/2}. \tag{22}$$

For $\rho' \ll \rho$ this expression coincides with (21). It follows from (19), (20), and (22) that for natural bubble ascent in the field of gravity we again have the relation $Sh = Sh_\infty$.

The foregoing results correspond to models in which the mutual impenetrability of spherical inclusions of the disperse phase is either ignored altogether or is described by the introduction of a homogeneous layer of pure liquid separating the "test" inclusion from the fictitious continuum. Unfortunately, the direct extension of these results to the more complex models treated in [14, 15], which correspond to a fictitious medium with strongly inhomogeneous properties, is impeded by the fact that the flow of this medium in its region of inhomogeneity near the surface of the test inclusion cannot be considered potential in general, as is readily verified on the basis of our previous results [12].

NOTATION

C, constant in Eq. (17); D, molecular diffusivity; d, density; E, uniform average-pressure gradient; **F**, force of hydrodynamic interaction; **F***, interaction force associated with viscous stresses; f, function in Eq. (5); G, volume flow rate of gas in a column; **g**, acceleration of gravity; K, coefficient in the expression for **F***; k, permeability; $n = \rho/V$; P, p, true and average pressures; **Q**, **q**, true and average filtration rates in a porous matrix or velocities of liquid between bubbles; R, radius of inclusion in a porous medium or of spherical bubbles or the frontal part of cap bubbles; S, function in Eqs. (3) and (4); s, coordinate along a streamline; U, average velocity of uniform flow; U_b, bubble rising velocity in a coordinate system in which the liquid is at rest on the average; V, bubble volume; w_s, tangential velocity in a surface layer; α, coefficient of ρ in the expansion of U_b/U_∞; β, function defined in [2]; γ, Péclet structure number lU/D; l, structure microscale; μ, fluid viscosity; ρ, volume concentration of inhomogeneities in a porous medium or of spheres comprising bubbles and their rotational wake domains; ρ', volume concentration of cap bubbles; θ, angular coordinate; $Pe = 2RU/D$; $Sh = 2RK_m/D$, Sherwood number; K_m, mass-transfer coefficient. Indices: 0, continuous phase; 1, discontinuous phase; ∞, infinitely diluted system ($\rho \to 0$).

LITERATURE CITED

1. V. G. Levich, Physicochemical Hydrodynamics, Prentice-Hall.
2. D. W. Moore, "The boundary layer on a spherical bubble," J. Fluid Mech., 16, 161-176 (1963).
3. W. Winnikow, "Comment on the paper 'Mass transfer and drag coefficients for single bubbles' by T. A. Redfield and G. Houghton," Chem. Eng. Sci., 22, 477 (1967).
4. R. M. Davies and G. I. Taylor, "The mechanics of large bubbles rising through extended liquids and through liquids in tubes," Proc. R. Soc. London, Ser. A, 200, 375-390 (1950).
5. J.-Y. Parlange, "Spherical cap bubbles with laminar wakes," J. Fluid Mech., 37, 257-263 (1969).
6. M. H. I. Baird and J. F. Davidson, "Gas absorption by large rising bubbles," Chem. Eng. Sci., 17, 87-93 (1962).
7. A. C. Lochiel and P. H. Calderbank, "Mass transfer in the continuous phase around axisymmetric bodies of revolution," Chem. Eng. Sci., 19, 471-494 (1964).
8. R. I. L. Guthrie, "Dynamics and mass transfer phenomena of spherical-capped bubbles," Ph.D. Dissertation, Univ. London, London (1967).
9. R. E. Sundell, "An experimental investigation of spherical cap bubbles in liquids," Ph.D. Dissertation, Yale Univ., New Haven, CT (1971).
10. G. Marrucci, "Rising velocity of a swarm of spherical bubbles," Ind. Eng. Chem. Fund., 4, 224-225 (1965).

11. B. P. Leclair and A. E. Hamielec, "Viscous flow through particle assemblages at intermediate Reynolds numbers — a cell model for transport in bubble swarms," Can. J. Chem. Eng., 49, 713-720 (1971).
12. Yu. A. Buevich (Buyevich) and I. N. Shchelchkova, "Flow of dense suspensions," Prog. Aerospace Sci., 18, 121-150 (1978).
13. R. Herczynski and I. Pienkowska, "Toward a statistical theory of suspensions," Ann. Rev. Fluid Mech., 12, 237-269 (1980).
14. Yu. A. Buevich and Yu. A. Korneev, "Effective thermal conductivity of a disperse medium," Inzh.-Fiz. Zh., 31, No. 4, 607-612 (1976).
15. B. S. Endler, "Effective viscosity and thermal conductivity of a disperse medium," Inzh.-Fiz. Zh., 37, No. 1, 110-117 (1979).
16. Yu. A. Buevich and V. G. Markov, "Steady-state transport in fibrous materials," Inzh.-Fiz. Zh., 36, No. 5, 828-834 (1979).
17. Yu. A. Buevich and D. A. Kazenin, "Limiting problems of heat or mass transfer toward a cylinder and a sphere immersed in an infiltrable granular layer," Zh. Prikl. Mekh. Tekh. Fiz., No. 5, 94-102 (1977).
18. G. K. Batchelor, Introduction to Fluid Dynamics, Cambridge Univ. Press (1967).
19. G. B. Wallis, One-Dimensional Two-Phase Flow, McGraw-Hill, New York (1969).
20. Yu. A. Buevich and I. N. Shchelchkova, "Rheological properties of homogeneous finely disperse suspensions: steady flows," Inzh.-Fiz. Zh., 33, No. 5, 872-879 (1977).
21. D. J. Nicklin, "Two-phase bubble flow," Chem. Eng. Sci., 17, 693-702 (1962).
22. Yu. A. Buevich and A. N. Deryabin, "Evolution of a steady inhomogeneous fluidization state for a uniform flow distribution of the fluidizing medium," Inzh.-Fiz. Zh., 36, No. 3, 416-425 (1979).
23. A. J. Bridge, L. Lapidus, and J. C. Elgin, "The mechanics of vertical gas—liquid fluidized systems. I. Countercurrent flow," AIChE J., 10, 819-826 (1964).

MASS TRANSFER BETWEEN THE GAS AND SOLID PARTICLES IN A FLUIDIZED BED

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UDC 662.62:66.096.5

A model of the boundary layer and condition of suspension of a solid particle by a gas flow is used to formulate analytical correlations, which are then compared with experiment.

Fluidization techniques have come increasingly under the scrutiny of power engineers in recent years in connection with the growing use of coal as a source of energy in the face of tightening environmental-protection demands.

Coal combustion takes place in a fluidized bed of coarsely disperse material, and the process is largely governed by mass transfer toward the surface of the burning particle. The mass transfer between the gas and particulate material in a fluidized bed has been studied under conditions such that all particles are involved in the transfer process. A wealth of experimental data has been accumulated to date and has been generalized in a survey paper [1]. Inasmuch as the concentration of coal in the fluidized bed is small, not more than 1-2%, the available information is unfortunately of only minor practical interest. An exception is the recently reported work of Hsiung and Thodos [2] on the sublimation of naphthalene in a fluidized bed of polymer particles. The authors have obtained some interesting experimental data, but they failed to comprehend the mass-transfer laws in the system. Our present objective is to bridge this gap and to investigate the mass transfer between the particles and the gas in a fluidized bed on the basis of reasonably general properties of the system, namely that:

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 41, No. 6, pp. 1067-1072, December, 1981. Original article submitted March 6, 1981.

 0022-0841/81/4106-1346$07.50